13 Solve

a $\sin x \cos x = 1 - 2.5\cos 2x$ in interval $[0, 360]$

$$\frac{1}{2}\sin 2x = 1 - \frac{5}{2}\cos 2x$$

$$\frac{1}{2}\sin 2x + \frac{5}{2}\cos 2x = 1$$

Put $R\sin(2x + \alpha) = \frac{1}{2}\sin 2x + \frac{5}{2}\cos 2x$

$$R\sin 2x\cos\alpha + R\cos 2x\sin\alpha = \frac{1}{2}\sin 2x + \frac{5}{2}\cos 2x$$

$\Rightarrow R\cos\alpha = \frac{1}{2}$

and $R\sin\alpha = \frac{5}{2}$

$\tan\alpha = 5 \qquad \alpha \simeq 78.7°$

$R = \frac{\sqrt{26}}{2}$

$$\frac{\sqrt{26}}{2}\sin(2x + 78.7) = 1$$

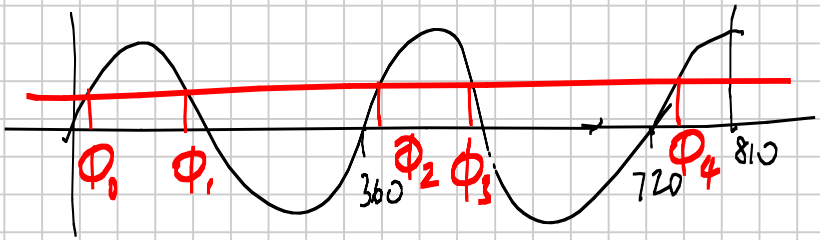
$$\sin(2x + 78.7) = \frac{2}{\sqrt{26}}$$

$0 \leq x \leq 360$

$0 \leq 2x \leq 720$

$78.7 \leq 2x + \alpha \leq 798.7$

$78.7 \leq \phi \leq 798.7$



$$\phi_0 = \sin^{-1}\left(\frac{2}{\sqrt{26}}\right)$$

$$x = \frac{\phi - \alpha}{2}$$

$\phi_0 = 23.1°$

$\phi_1 = 156.9°$

$\phi_2 = 383.1°$

$\phi_3 = 516.9°$

$\phi_4 = 743.1°$

$x_1 = 39.1°$

$x_2 = 152.2°$

$x_3 = 219.1°$

$x_4 = 332.2°$

13b Solve $\cot\theta + 2 = \operatorname{cosec}\theta$ $[0 < \theta < 360]$ where $\theta \neq 180°$.

$$\frac{\cos\theta}{\sin\theta} + 2 = \frac{1}{\sin\theta}$$

$$\cos\theta + 2\sin\theta = 1$$

put $R\sin(\theta+\alpha) \equiv \cos\theta + 2\sin\theta$

$\Rightarrow$ $R\sin\theta\cos\alpha + R\cos\theta\sin\alpha \equiv \cos\theta + 2\sin\theta$

$\Rightarrow$ $R\cos\alpha = 2$

$R\sin\alpha = 1$

$\tan\alpha = \frac{1}{2}$ $\Rightarrow \alpha = 26.56505117\ldots \simeq 26.6°$

$R^2\sin^2\alpha + R^2\cos^2\alpha = R^2 = 1^2 + 2^2$

$\Rightarrow R = \sqrt{5}$.

13b ctd.

$$\sqrt{5}\,\sin(\theta + 26.6^\circ) = 1$$

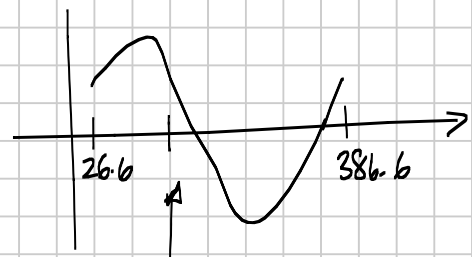
$$\Rightarrow \sin(\theta + 26.6) = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \theta + 26.6 = 153.43\ldots$$

$$\theta = 126.869\ldots$$

$$\theta \simeq 126.9^\circ$$

$26.6 < \theta + 26.6 < 386.6$



$180 - 26.6$
$\simeq 153.4^\circ$

only solution since $0 < \theta < 360$

strict inequality

13c

$$\sin\theta = 2\cos\theta - \sec\theta$$

$$\sin\theta\cos\theta = 2\cos^2\theta - 1$$

$$\tfrac{1}{2}\sin 2\theta = \cos 2\theta$$

$$\tan 2\theta = 2$$

$$\tan\phi = 2$$

$$\Rightarrow \quad \phi_1 = 63.434948822\ldots^\circ$$

$$\phi_2 = 243.434948822\ldots^\circ$$

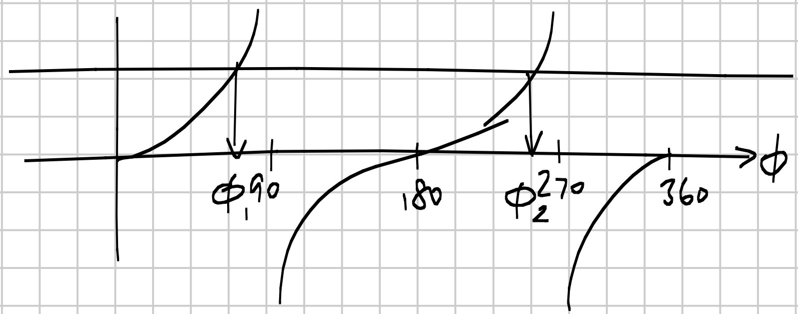
$$\theta_1 \simeq 31.7^\circ$$

$$\theta_2 \simeq 121.7^\circ$$

$[0, 180^\circ]$

$$0 \leq \theta \leq 180$$

$$\Rightarrow \quad 0 \leq \phi \leq 360$$



13d $\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) + (\sqrt{3}-1)\sin\theta = 2$ in the interval $0 \leq \theta \leq 2\pi$

$\sqrt{2}\cos\theta\cos\left(\frac{\pi}{4}\right) + \sqrt{2}\sin\theta\sin\left(\frac{\pi}{4}\right) + \sqrt{3}\sin\theta - \sin\theta = 2$

since $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

we have $\cancel{\sqrt{2}}\cos\theta\left(\cancel{\frac{1}{\sqrt{2}}}\right) + \cancel{\sqrt{2}}\sin\theta\left(\cancel{\frac{1}{\sqrt{2}}}\right) + \sqrt{3}\sin\theta - \sin\theta = 2$

$\Rightarrow \quad \cos\theta + \sqrt{3}\sin\theta = 2$

put $\cos\theta + \sqrt{3}\sin\theta = R\sin(\theta + \alpha)$

then $\cos\theta + \sqrt{3}\sin\theta = R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$

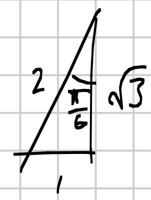
$\Rightarrow \quad R\cos\alpha = \sqrt{3}$

and $R\sin\alpha = 1$

$\Rightarrow R^2\left(\cancel{\sin^2\alpha + \cos^2\alpha}\right) = 1 + 3$

$\Rightarrow R = 2$

while $\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \frac{1}{\sqrt{3}} \Rightarrow \alpha = \frac{\pi}{6}$



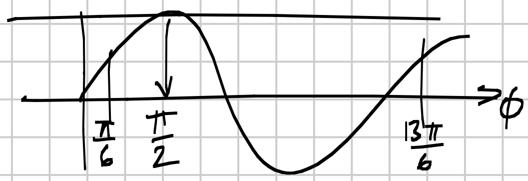
13d

Hence

$$\sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right) + \left(\sqrt{3}-1\right)\sin\theta = 2$$

$$\Rightarrow \quad 2\sin\left(\theta + \frac{\pi}{6}\right) = 2$$

define $\phi \equiv \theta + \frac{\pi}{6}$ & solve $\sin\phi = 1$ in the interval $\frac{\pi}{6} \leq \phi \leq \frac{13\pi}{6}$



$$\Rightarrow \quad \phi = \frac{\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}.$$